This packet of information on

Dyscalculia (Math)

is comprised of the following articles from *Perspectives* (P), our quarterly journal for members of IDA:

Mathematical Overview: An Overview for Educators
David C. Geary
(P: Vol. 26, No. 3, Summer 2000, p. 6-9)

Mathematical Learning Profiles and Differentiated Teaching Strategies
Maria R. Marolda, Patricia S. Davidson
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Translating Lessons from Research into Mathematics Classroom
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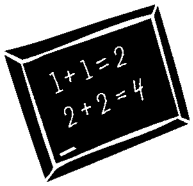

MATHEMATICAL DISORDERS: AN OVERVIEW FOR EDUCATORS

By David C. Geary

During the past 20 years, we have witnessed enormous advances in our understanding of the genetic, neurological, and cognitive factors that contribute to reading disorders, as well as advances in the ability to diagnose and remediate this form of learning disorder (e.g., Torgesen et al., 1999). We now understand that most forms of reading disorder result from a heritable risk and have a phonological core; for instance, many of these children have difficulties associating letters and words with the associated sounds, which makes learning to decode unfamiliar words difficult (Light, DeFries, & Olson, 1998). At the same time, there have been a handful of researchers studying children's difficulties with early mathematics, difficulties that emerge despite low-average or better intelligence and adequate instruction (Geary, Hamson, & Hoard, in press; Jordan & Montani, 1997). This essay overviews this research, including discussion of the prevalence of children with mathematical disorders and their diagnoses, the approach researchers use to study these children, and some major findings.

How common is a Mathematical Disorder and how is it diagnosed?

Although there are no definitive answers, studies conducted in the United States, Europe, and Israel all converge on the same conclusion: About 6% of school-age children and adolescents have some form of mathematical disorder and about one half of these individuals also have difficulty in learning how to read (Gross-Tsur, Manor, & Shalev, 1996). These studies also suggest that mathematical disorders are as common as reading disorders and that a common deficit may contribute to the co-occurrence of a mathematical disorder and a reading disorder in some children (Geary, 1993).

Like reading disorders, there is no universally agreed upon set of criteria for the diagnosis of mathematical disorders. In our recent work, we have found a lower than expected (based on IQ) performance on math achievement tests across at least two grade levels to be a useful and practical indicator of a mathematical disorder (Geary et al., in press). This and other studies indicate that children with a mathematical disorder are a heterogeneous group and show one or more subtypes of disorder (Geary, 1993).

How do researchers approach the study of Mathematical Disorders?

The complexity of the field of mathematics makes the study of mathematical disorders challenging. In theory, mathematical disorders can result from deficits in the ability to represent or process information in one or all mathematical domains or in one or a set of individual competencies within each domain. As an example, the assessment of computational skills in dyscalculia (poor performance after brain injury) has often been based on summary scores for accuracy at solving simple (e.g. 9+6) and complex (e.g. 244+129) arithmetic problems (Geary, 1993). These scores actually reflect an array of component skills, including fact retrieval and procedural as well as conceptual competencies, making inferences about the source of poor performance on these arithmetic problems imprecise. Moreover, in geometry and algebra, not enough is known about the normal development of the associated competencies to provide a systematic framework for the study of mathematical disorders. Fortunately, enough is now known about normal development in the areas of number concepts, counting skills, and early arithmetic skills to provide the framework needed

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Mathematical Disorders: Subtype 1 Semantic Memory	**Mathematical Disorders: Subtype 2 Procedural**	**Mathematical Disorders: Subtype 3 Visuospatial**
Cognitive & Performance Features: A. Low frequency of arithmetic fact retrieval B. When facts are retrieved, there is a high error rate C. Errors are frequently associates of the numbers in the problem D. Solution times for correct retrieval are unsystematic	*Cognitive & Performance Features:* A. Relatively frequent use of developmentally immature procedures B. Frequent errors in the execution of procedures C. Potential developmental delay in the understanding of the concepts underlying procedural use D. Difficulties sequencing the multiple steps in complex procedures	*Cognitive & Performance Features:* A. Difficulties in spatially representing numerical information such as the misalignment of numerals in multicolumn arithmetic problems or rotating numbers B. Misinterpretation of spatially represented numerical information, such as place value errors C. May result in difficulties in areas that rely on spatial abilities, such as geometry
Neuropsychological Features: A. Appears to be associated with left hemispheric dysfunction, in particular, posterior regions of the left hemisphere B. Possible subcortical involvement, such as the basal ganglia	*Neuropsychological Features:* Unclear, although some data suggest an association with left hemispheric dysfunction, and in some cases a prefrontal dysfunction	*Neuropsychological Features:* Appears to be associated with right hemispheric dysfunction, in particular, posterior regions of the right hemisphere, although the parietal cortex of the left hemisphere may be implicated as well.
Genetic Features: Preliminary studies and the relation with certain forms of reading disorder suggest that this deficit may be heritable	*Genetic Features:* Unclear	*Genetic Features:* Unclear
Relationship to Reading Disorders: Appears to occur with phonetic forms of reading disorder	*Relationship to Reading Disorders:* Unclear	*Relationship to Reading Disorders:* Does not appear to be related

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to systematically study mathematical disorders (see Geary, 1994, for a review). The sections below provide an overview of what we now know about children's developing number concepts, counting skills, and early arithmetic skills along with a discussion of any associated learning disorder. The final section provides a discussion of visuospatial issues sometimes associated with children with mathematical disorders.

Number Concepts

Psychologists have been studying children's conceptual understanding of number, for instance that "3" is an abstract representation of a collection of any three things, for many decades. It is now clear children's understanding of small quantities and number is evident to some degree in infancy. Their understanding of larger numbers and related skills, such as place value concepts (e.g., the "4" in the numeral "42" represents four groups of 10), emerges slowly during the preschool and early elementary school years and some times only with instruction (Fuson, 1988; Geary, 1994).

The few studies conducted with children with mathematical disorders suggest that basic number competencies, at least for small quantities, are intact in most of these children (Geary, 1993; Gross-Tsur et al., 1996).

Counting Skills

During the preschool years, children's counting knowledge can be represented by Gelman and Gallistel's (1978) five implicit counting principles. These principles include one-one correspondence (one and only one word tag, such as "one," "two," is assigned to each counted object); the stable order principle (the order of the word tags must be invariant across counted sets); the cardinality principle (the value of the final word tag represents the quantity of items in the counted set); the abstraction principle (objects of any kind can be collected together and counted); and, the order-irrelevance principle (items within a given set can be tagged in any sequence). Children also make seemingly apparent, but not necessarily correct, inductions about the basic characteristics of counting by observing standard counting behavior (Briars & Siegler, 1984; Fuson, 1988). These inductions include "adjacency" (counting must proceed consecutively and in order from one to the next) and "start at an end" (counting must proceed from left to right).

Studies of children with concurrent mathematical disorders and reading disorders or mathematical disorders alone indicate that these children understand most of the essential features of counting, such as stable order, but consistently err on tasks that assess "adjacency" and order-irrelevance (Geary, Bow-Thomas, & Yao, 1992; Geary et al., in press). In fact, these children, at least in first and second grade, perform more poorly on these tasks than do children with much lower IQ scores, suggesting a very specific deficit in their counting knowledge. It appears that these children, regardless of their reading achievement, believe that counting is constrained such that counting procedures can only be executed in the standard way (i.e., objects can only be counted sequentially), which, in turn, suggests that they do not fully understand counting concepts.

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> *Difficulties in using counting procedures can thus contribute to later arithmetic-fact retrieval problems.*

Other studies suggest that children with mathematical disorders also have difficulties keeping information in working memory while monitoring the counting process or performing other mental manipulations (Hitch & McAuley, 1991), which, in turn, results in more errors while counting. Thus, young children with mathematical disorders show deficits in counting knowledge and counting accuracy.

Arithmetic Skills

When first learning to solve simple arithmetic problems (e.g., 3+5), children typically rely on their knowledge of counting and use counting procedures to find the answer (Geary et al., 1992; Siegler & Shrager, 1984). Their procedures sometimes rely on finger counting and sometimes only on verbal counting. Common counting procedures include the following: *sum (or counting-all)*, where children count each addend starting from 1; *max (or counting on)*, where children state the value of the smaller addend and then count the larger addend; and, *min (counting on)*, where children state the larger addend and then count the value of the smaller addend, such as stating 5 and counting on 6, 7, 8 to solve 3+5.

The development of efficient counting procedures for simple problems (e.g., 3+5) reflects a gradual shift from frequent use of the sum and max procedures to frequent use of the min procedure. The repeated use of counting procedures also appears to result in the development of memory representations of basic facts (Siegler & Shrager, 1984), that is, with repeated counting the generated answer (e.g., 8) eventually becomes associated in memory with the problem (e.g., 3+5). Difficulties in using counting procedures can thus contribute to later arithmetic-fact retrieval problems.

Studies conducted in the United Stâtes, Europe, and Israel have consistently found that children with mathematical disorders have difficulties solving simple and complex arithmetic problems (e.g., Barrouillet, Fayol, & Lathulìere, 1997; Jordan & Montani, 1997). These differences involve both procedural and memory-based deficits, each of which is considered in the respective sections below.

Procedural Deficits

Much of the research on children with mathematical disorders has focused on their use of counting strategies to solve simple arithmetic problems and indicates that these children commit more errors than do their normal peers (Geary, 1993; Jordan & Montani, 1997). They often miscount or lose track of the counting process. As a group, young children with mathematical disorders also rely on finger counting and use the sum procedure more frequently than do

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normal children. Their use of finger counting appears to be a working memory aid, in that it helps these children to keep track of the counting process. Their prolonged use of sum counting appears to be related, in part, to their belief that "adjacency" is an essential feature of counting (Geary et al., 1992). However, many, but not all, of these children show more efficient procedures by the middle of the elementary school years (Grades 4-6) (Geary, 1993; Jordan & Montani, 1997). Thus, for these children, their error-prone use of immature procedures represents a developmental delay rather than a long-term cognitive deficit.

There have only been a few studies of the ability of children with mathematical disorders to pursue formal arithmetic algorithms associated with more complex problems, such as 126+537. The research that has been conducted suggests some specific difficulties. Although some studies have attributed calculation difficulties to visuospatial difficulties described below, other studies suggest that these calculation difficulties are likely not due to the spatial demands of these arithmetic formats, as most children with mathematical disorders do not have poor spatial abilities (e.g., Geary et al., in press). Rather, the errors appeared to result from difficulties in monitoring the sequence of steps of the algorithm and from poor skill in detecting and then self-correcting errors. Thus, procedural difficulties associated with mathematical disorders are evident when these children count to solve simple arithmetic problems (e.g., 3+5) and use algorithms to solve more complex problems (e.g., 126 + 537).

Retrieval Deficits

Many children with mathematical disorders do not show the shift from direct counting procedures to memory-based production of solutions to simple arithmetic problems that is commonly found in normal children. It appears that there are two different forms of retrieval deficit, each reflecting a disruption to different cognitive and neural systems (Barrouillet et al., 1997; Geary, 1993).

Cognitive studies suggest that the retrieval deficits are due, in part, to difficulties in accessing facts from long-term memory. In fact, it appears that the memory representations for arithmetic facts are supported, in part, by the same phonological and semantic memory systems that support word decoding and reading comprehension. If this is indeed the case, then the disrupted phonological processes that contribute to reading disorders might also be the source of the fact retrieval difficulties of children with mathematical disorders. It might be the source of the co-occurrence of mathematical disorders and reading disorders in many children (Geary, 1993; Light et al., 1998).

Recent studies suggest a second form of retrieval deficit, specifically, disruptions in the retrieval process due to difficulties in inhibiting the retrieval of irrelevant associations. This form of retrieval deficit was discovered by Barrouillet et al. (1997) and was recently confirmed (Geary et al., in press). In the latter study, first and second grade children with concurrent mathematical disorders and reading disorders, mathematics disorders alone, or reading disorders alone were compared to their normal peers. On one of the tasks, the children were instructed not to use counting procedures but only use retrieval techniques to find solutions for simple addition problems. Children in all learning disorder groups committed more retrieval errors than their normal peers did, even after controlling for IQ. The most common error was a counting string associate of one of the addends. For instance, common retrieval errors for the problem 6+2 were 7 and 3, the numbers following 6 and 2, respectively, in the counting sequence. The pattern across studies suggests that inefficient inhibition of irrelevant associations contributes to the retrieval difficulties of children with mathematical disorders. The solution process is efficient when irrelevant associations are inhibited and prevented from entering working memory. Insufficient inhibition results in activation of irrelevant information, which functionally lowers working memory capacity. In this view, children with mathematical disorders may make retrieval errors because they cannot inhibit irrelevant associations from entering working memory. Once in working memory these associations either suppress or compete with the correct association (i.e., the correct answer) for expression.

Disruptions in the ability to retrieve basic facts from long-term memory, whether the cause is accessing difficulties or the lack of inhibition of irrelevant associations, might, in fact, be considered a defining feature of mathematical disorders (Geary, 1993). Moreover, characteristics of these retrieval deficits (e.g., solution times) suggest that for many children these do not reflect a simple developmental delay but rather a more persistent cognitive disorder.

Visuospatial Skills

In a variety of neuropsychological studies, specific difficulties with visuospatial skills have been associated with dyscalculia, with specific reference to spatial acalculia. The particular features associated with spatial acalculia include the misalignment of numerals in multi-column arithmetic problems, numeral omissions, numeral rotation, misreading arithmetical operation signs and difficulties with place value and decimals (see Geary 1993). Russell and Ginsburg (1984) found that fourth-grade children with mathematical disorders committed more errors than their IQ-matched normal peers on complex arithmetic problems (e.g., 34x28). These errors involved the misalignment of numbers while writing down partial answers or errors while carrying or borrowing from one column to the next. The children with mathematical disorders appeared to understand the base-10 system as well as the normal children did, and thus the errors could not be attributed to a poor conceptual understanding of the structure of the problems (see also Rourke & Finlayson, 1978). Other studies suggest that spatial deficits will also influence the ability to solve other types of mathematics problems, such as word problems and certain types of geometry problems (Geary, 1996). In elementary school, however, this subtype of mathematical disorder does not appear to be as common as the other subtypes.

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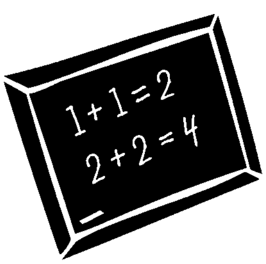
Conclusion

As a group, children with mathematical disorders show a normal understanding of number concepts, concepts underlying arithmetic algorithms, and most counting principles. At the same time, many of these children have difficulty keeping information in working memory while monitoring the counting process and seem to understand counting only as a rote, mechanical process (i.e., counting can only proceed with objects counted in a fixed order). When solving simple arithmetic problems, young children with mathematical disorders use developmentally immature procedures and commit many more errors in the execution of these procedures. Since many of these children eventually develop efficient counting procedures, their difficulties in this area represent a developmental delay. A defining feature of mathematical disorders that does not appear to improve with age or schooling is difficulty retrieving basic arithmetic facts from long-term memory. This memory deficit appears to result from a more general difficulty in representing information in or retrieving information from phonetic and semantic memory, as well as from difficulties in inhibiting the retrieval of irrelevant associations. Finally, many children with mathematical disorders have difficulties in organizing the sequence of steps needed to successfully pursue formal algorithms. Future studies will, no doubt, clarify these patterns and lay the foundation for remedial strategies.



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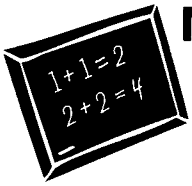

MATHEMATICAL LEARNING PROFILES AND DIFFERENTIATED TEACHING STRATEGIES

By Maria R. Marolda and Patricia S. Davidson

Are there particular mathematical profiles that characterize how students learn mathematics?

How does the understanding of mathematical learning profiles translate into better instrutional opportunities in mathematics?

A primary consideration in the teaching of mathematics is the recognition that students bring to the mathematics classroom a wide range of abilities and learning approaches. Extensive instructional and clinical investigations during the past 20 years, as well as a detailed research study (Davidson, 1983), have revealed that students' learning profiles are marked by different constellations of relative strengths and relative weaknesses with which students face the world of mathematics. Indeed, it is this study of *differences*, rather than a definition of explicit deficits, that provides a more useful approach to understanding students' effectiveness or inefficiencies in learning mathematics. Moreover, an understanding of differences is also informative in fashioning instructional approaches that are compatible with the various learning profiles that exist in the mathematics classroom.

Child / World System

Learning profiles in mathematics can best be understood by considering a *Child/World* system (Bernstein & Waber, 1990) that characterizes the reciprocal relationship of the developing child and the mathematical world in which the child must function. The construction of a *Child/World* system focusses on differences among learners as well as differences in the demands of what is to be learned. It then explores instances of *match* and *mismatch.* In the consideration of differences among students, the critical question becomes, "When does a learning difference render a student learning disabled?" A learning "disability" in mathematics may be thought of as the occurrence of multiple "mismatches" and the inability to overcome those mismatches. A tantalizing issue then becomes whether specific approaches or strategies could be used so that the mismatches are minimized and the disability is resolved or disappears.

It is important to recognize that the diagnostic process in education is quite different from the diagnostic process in medicine. Whereas the medical diagnostician is looking to uncover what is wrong and what the patient can't do, the educator must strive to uncover the student's strengths and what the student can do. The goal of the educator is to find those strengths that can be used to address the weaknesses and difficulties inherent in students' learning profiles.

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> *The construction of a Child/World system focuses on differences among learners as well as differences in the demands of what is to be learned.*

In focussing on the *Child* in the *Child/World* system of mathematics, a multidimensional view must be taken and a variety of parameters considered. Specifically, the following factors should be explored in order to understand a student's Mathematical Learning Profile:

- the presence of specific developmental features that are prerequisite to specific mathematics topics;
- the preferred models with which mathematical topics are interpreted;
- the preferred approaches with which mathematical topics are pursued;
- memory skills that affect students' ability to participate in mathematical activities;
- language skills that affect students' ability to participate in the mathematical arena.

Developmental Features of Mathematical Learning Profiles

A definition of a student's mathematical learning profile should incorporate an appreciation of the developmental maturity of students at various ages. There are many developmental milestones in terms of mathematical readiness for dealing with numerical, spatial and logical topics. For numerical concepts, the developmental milestones consist of an appreciation of number, the concept of number (enumeration/cardinality), conservation of number, one to one correspondence and the principles of class inclusion. For spatial concepts, the construct of space, conservation of length and conservation of volume must be considered. For logical thought, developmental milestones include the concepts underlying classification, seriation, associativity, reversibility and inference. Most children between the ages of four and eight have acquired these milestones.

Recent clinical investigation and teaching practice have suggested that the concept of place value might also be developmentally mediated (Marolda & Davidson, 1994). That is, an appreciation of place value depends more on the state/age of the child than on specific teaching experiences. If the child is not cognitively ready to deal with place value, then the concept of place value cannot be formally or meaningfully developed, despite teaching efforts. The formal concept of place value seems to be established for most children between the ages of six and eight. The appreciation of formal place value concepts is of particular importance since they are necessary prerequisites for the understanding of larger quantities and the pursuit of multi-digit computation.

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Preferred Models and Preferred Approaches of Mathematical Learning Profiles

Mathematical situations can be interpreted with concrete, pictorial, or symbolic models. For a particular student, a specific interpretation might be more comfortable and meaningful. Among concrete models, further distinctions can be made. Within the concrete mode, students may prefer set (discrete) models, such as counters, while others appreciate perceptually driven (measurement) models, such as Cuisenaire rods.

The ways in which students process or approach mathematical situations follow two distinct patterns (Marolda & Davidson, 1994). Some students process situations in a linear fashion, building forward to an exact final solution. Sometimes, these students are so focused on the individual elements that the overall thrust or goal is obscured. This style of processing is often characterized as a sequential, step-by-step approach. For other students, a careful building up approach holds little inherent meaning. Such students prefer to establish a general overview of a situation first and then refine that overview successively until an exact solution emerges. Such students may be prone to imprecision and tend to lack appreciation of all relevant details. This style of processing is often described as global or gestalt.

Incorporating these inherent preferences in terms of models and processing has led to the definition of two distinct learning profiles in mathematics, *Mathematics Learning Style I* and *Mathematics Learning Style II* as reviewed in Table 1 (Marolda & Davidson, 1994). Moreover, it is possible to describe mathematical concepts and procedures that are inherently compatible with each learning profile.

To be full and successful participants in mathematics, students must learn to mobilize both Mathematics Learning Style I and Mathematics Learning Style II. The student with special learning needs, however, is often limited to one learning style alone and is unable to mobilize skills and strategies associated with the alternative learning style. For success, teachers must translate activities into the student's operating style, building a scaffold that integrates the areas of strengths and weaknesses so that they complement one another and lead to the acquisition of mathematical concepts and procedures in a meaningful way.

Mathematics Learning Style I	Mathematics Learning Style II
Preferred Models for Number: • Set Models	*Preferred Models for Number:* • Perceptual (Measurement) Models
Preferred Approaches: • Linear, step by step • Often relies on verbal mediation	*Preferred Approaches:* • Deductive, global • Often relies on successive approximations
Topics Approached with Ease: • Counting forward & counting-on • Concepts of addition & multiplication • Pursuit of calculation procedures • Fraction concepts interpreted in verbal terms • Geometric Shapes: Emphasis on naming	*Topics Approached with Ease:* • Counting backward • Concepts of subtraction & division • Estimation • Fraction concepts interpreted in a variety of visual models • Geometric Shapes: Emphasis on spatial relationships and manipulations
Topics of Particular Challenge: • Broader concepts and overarching principles • Estimation strategies • Appreciation of appropriateness of solution generated • Selection of arithmetic operation in word problems; difficulty switching between operations in a set of word problems • Concept of a fraction • More sophisticated geometric topics • Requirement for flexible or alternative approaches	*Topics of Particular Challenge:* • Appreciation of all salient details of multi-step procedures or word problems • Pursuit of multi-step calculation procedures • Relevance of exact solutions; prefers to guess • Follow through to exact solutions in word problems, despite correct choice of operation • Formal fraction operations, despite comfort with underlying fraction concept • Requirement to describe approach in exacting verbal terms • Insistence on a single, specific approach

Table 1

The following charts (Tables 2 & 3; as shown on pages 13 and 14) offer more explicit features of each of the Mathematical Learning Styles and can be helpful in recognizing them and teaching to them.

Memory Skills as a Feature of Mathematical Learning Profiles

Often students are characterized as having difficulty in mathematics because they "can't remember." The attribution of mathematical difficulties to a global memory deficit is somewhat simplistic. Cognitive psychologists (Holmes, 1988) suggest that memory issues, in general, are very complex.

> In evaluating a child's recall of materials, the clinician should recognize the various components of the process loosely called memory: registration of the stimulus, encoding, organization, storage and retrieval....Learning disabled children, however, are constantly described in the psychological and educational literature as having memory deficits of various types, usually visual or auditory (short term or otherwise). In almost all cases, the impairment involves either the initial encoding or the effective retrieval of information. (p.189)

In mathematics, it is particularly important to consider the distinction between encoding and retrieval aspects of memory. Is the student having difficulty remembering the fact or procedure because it was never properly understood and therefore not encoded for storage in memory? Or is the student having difficulty remembering the fact or procedure because it cannot be accessed from the student's repertoire of learned skills?

Four specific memory skills are important in mathematics:

- retrieval of solutions to one digit facts;
- the recall of the sequence of multi-step procedures;

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- visual memory of perceptual / geometric stimuli;
- recall of mathematical data presented auditorially.

In terms of retrieval difficulties in the production of solutions to one digit facts, mathematically it may be more important to consider if the solution is produced efficiently rather than automatically. The distinction that is important is whether the retrieval is automatic or efficient. Difficulties with the retrieval of one digit facts may be supported by alternative strategies that are compatible with a student's inherent learning style and result in more efficient production of solutions. In the example, 8 + 6, a student with Mathematics Learning Style I would be most efficient turning to counting on strategies: 9,10,11,12, 13...**14**! or strategies that build 10s: 8 + (2+4) = 10 + 4 = **14**! A student with Mathematics Learning Style II would be most efficient turning to related facts, e.g. doubles, 8+8=16, so 8+6=14 ...2 Less! Or 6+6=12, so 8+6=14...2 More!

In dealing with multi-step procedures, the recall of the organization of the specific steps relies on an understanding of the conceptual foundations driving the procedure. By offering alternative approaches that appeal to a specific learning style, the procedure is better understood and more easily pursued. In dealing with the multiplication problem 23 x 14, a student with Mathematics Learning Style I would turn to a successive addition approach or the formal algorithm. Further supports to remembering the steps of procedures include encouraging verbal mediation techniques, developing verbal and visual flow charts that can be used as referents, and developing mnemonics to cue each step. In contrast, the student with Mathematics Learning Style II would turn to the definition of multiplication as an area and would then combine the area of the four subregions to determine the final solution. Further supports would be estimation techniques made iteratively or, once an initial estimate is made, the use of a calculator for an exact solution. With firm understanding established, the procedure is more effectively encoded. That understanding, however, may emerge from different approaches.

In dealing with geometric designs, students need to use visual memory skills. With visual memory difficulties, students may find the building and copying of geometric designs challenging. To support visual memory difficulties students might be encouraged to interpret geometric designs in verbal terms. Difficulties in visual memory can also manifest themselves in non-geometric situations, such as difficulties orienting written digits, difficulties aligning numerals in written procedures, and difficulty organizing a page of problems. Copying problems from the text and the board or interpreting data presented on a computer screen may also be difficult. In response, copying requirements should be minimized, while graphic organizers may be offered to support the copying that is required.

Students with auditory memory difficulties are challenged when required to remember all relevant data presented in instruction, remember the overall outcome sought, remember directions, or remember all the relevant information in word problem situations presented verbally. These students may be supported by offering directions in visual formats as well as by offering written directions and / or allowing students to write down the directions and then referring to the written text as needed. Interestingly students with apparent auditory memory issues are often confused with students whose primary difficulties are in language where memory difficulties are secondary to specific language processing issues.

Language Issues

Language skills, both oral and written, are important in mathematics in terms of:

- word retrieval skills;
- verbal formulation requirements;
- comprehension requirements.

They become an issue when students are required to retrieve the names of coins, geometric shapes or other mathematical terms, when they are asked to explain their solutions or approaches, when they must deal with lengthy verbal presentations typical of classroom instruction, and when they are faced with word problems. These language demands have become more prominent in mathematics as education curricula and textbooks have encouraged teachers to ask students for explanations or justifications of their approaches. Moreover, teachers have been encouraged to ask students to take responsibility for their own learning by reading printed materials or texts. These newer emphases pose particular challenges for students with language difficulties.

In order to address word retrieval difficulties in mathematics, students might focus primarily on the values of the coins rather than their specific names, might be encouraged to draw geometric shapes rather than name them and might be offered recognition formats when dealing with mathematical definitions. Retrieval issues are further supported by minimizing confrontational, fast answer situations.

Students with verbal formulation issues often have difficulty describing their approaches or in portfolio work where approaches must be written down. To support these students, alternate forms of communication should be encouraged, including demonstrations with physical models and use of pictures or diagrams to describe solution processes.

Students with comprehension difficulties often have difficulties with directions and with reading texts or word problems. They often can't get started with classwork, mistakenly suggesting they have attentional difficulties. These students benefit from careful monitoring of new presentations and having word problems read to them. Such students can be supported by teachers presenting content in meaningful "chunks" that are then carefully linked together. In terms of word problems, the situations can be presented verbally rather than requiring reading. Students should then be encouraged to draw pictorial interpretations to represent the situation and data involved.

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Mathematical Learning Profiles and Differentiated Teaching Strategies

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Mathematics Learning Style I		
Cognitive & Behavioral Correlates	**Mathematical Behaviors**	**Teaching Implications & Strategies**
• Highly reliant on verbal skills	• Approaches situations using recipes; "talks through" tasks • Interprets geometric designs verbally	• Emphasize the meaning of each concept or procedure in verbal terms. • Build on subvocalization strategies to direct procedures.
• Tends to focus on individual details or single aspects of a situation • Sees the "trees," but overlooks the "forest"	• Approaches mathematics in a mechanical, routine based fashion • Overwhelmed in situations in which there are multiple considerations, such as in multi-step tasks • Can generate correct solutions, but may not recognize when solutions are inappropriate • Difficulties "checking" work; must re-do entire problem • Difficulties choosing an approach in word problems • Difficulties appreciating larger geometric constructs because of an emphasis on component parts	• Highlight concept /overall goal. • Break down complex tasks into salient units and make linkage between units explicit. • Build simple estimation strategies; encourage two final steps to each calculation problem: "Does this answer the question?" and "Does the solution seem right?" • Encourage students to rewrite or state problems in their own words. • Develop metacognitive strategies to analyze word problem situations. • Encourage parts to wholes approach in building geometric figures and explicit descriptions of the overall design that emerges.
• Prefers HOW to WHY	• Prefers numerical approach over manipulative models • Needs drill and practice to establish procedure before considering applications or broader conceptual meaning	• Link manipulative model on a step-by-step basis to the numerical procedure. • Once procedure is secure, relate math topics to relevant real life situations.
• Relies on a defined sequence of steps to pursue a goal • Reliant on teacher for THE approach • Lack of versatility	• Prefers explicit delineation of each step of a procedure and linkage of steps one to another • Vulnerable when there are multiple approaches to a single topic • Overwhelmed by multiple models or multiple approaches • Prefers linear approaches for arithmetic topics	• Offer flow chart approaches. • Help students create handbooks with procedures described in their own words. • Choose one manipulative model or approach to develop a wide range of topics; avoid switching models or approaches too quickly. • Don't emphasize special cases; rather develop an over-riding rule that applies to all cases; e.g. for the addition and subtraction of fractions with unlike denominators, develop a single process using the product of the denominators in all cases, even if it is not the least common denominator. • Give explanations before or after procedure, but not while student is pursuing procedure. • Use counting on techniques for addition facts and missing addend techniques for subtraction facts. • Interpret multiplication as successive additions.
• Challenged by perceptual demands	• Difficulties with more sophisticated perceptual models, such as Cuisenaire rods • Geometric activities may be challenging, especially in three dimensions. • Difficulties interpreting analog clocks • Difficulties distinguishing coins, especially nickel and quarter • Difficulties organizing written formats	• Emphasize set (discrete) models for counting, such as money or counting chips. • Translate perceptual cues in terms of verbal descriptions.
• Prefers quizzes or unit tests to more comprehensive final exams	• May be able to complete the most difficult example in a set of examples relying on the same concept/skill, but has difficulty switching to a new topic or new approach	• Spiral all topics to keep them current.

Table 2

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Mathematical Learning Profiles and Differentiated Teaching Strategies

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Mathematics Learning Style II		
Cognitive & Behavioral Correlates	**Mathematical Behaviors**	**Teaching Implications & Strategies**
• Prefers perceptual stimuli and often reinterprets abstract situations visually or pictorially	• Benefits from manipulatives • Loves geometric topics	• Offer a variety of models; introduce perceptual models, such as *Base Ten Blocks* or *Cuisenaire Rods*, to support calculations. • Emphasize geometry as a vital part of the curriculum.
• Likes to deal with big ideas; doesn't want to be bothered with details	• Prefers concepts to algorithms • Tolerates ambiguity and imprecision • Offers impulsive guesses as solutions • Uses estimation strategies spontaneously • Skims word problems first but must be encouraged to re-read for salient details • Perceives overall shape of geometric configurations at the expense of an appreciation of the individual components	• Relate manipulative models to procedures before practicing algorithms. • Reward approach as well as precise solutions. • Develop an appreciation of how much precision a situation warrants. • Reward/encourage estimation strategies as first step. • Encourage diagrams as a technique to organize data in problem solving situations. • Allow calculators to support problem solving. • Encourage multiple refinements when building geometric designs in order to incorporate all the individual parts.
• Prefers WHY to HOW	• Requires a definition of overview before dealing with exacting procedures • Requires manipulative modeling before developing a concept or algorithm • Likes to set up problems, but resists following through to a conclusion	• Offer opportunities to work in cooperative groups.
• Prefers nonsequential approaches, involving patterns and interrelationships	• Prefers successive approximations approach to formal algorithms • Addition and multiplication facts involving 9s more readily generated because of underlying patterns that are recognized but not verbalized • Not troubled by mixed practice worksheets • Comfortable with horizontal formats for long calculations • Can offer a variety of alternative answers or approaches to a single problem • Can appreciate operation needed in a word problem but has difficulty following through to an exact solution • Likes logical problem solving in the form of general reasoning problems	• Allow alternative calculation procedures. • Help students to create their own handbooks of typical problems. • Generate arithmetic facts through relationships to known facts; e.g. doubles for + facts. • Emphasize area model for multiplication. • Start with real-life situation and tease out more formal arithmetic topics. • Use simulations, relating similar concepts/approaches in a variety of different situations. • Model complex problems with similar problems in simpler forms. • Give two grades on word problem activities; one for correct approach; one for exact final solution. • Include general reasoning examples in logical problem solving activities.
• Challenged by demands for details or the requirement of precise solutions	• Difficulties with precise calculations • Difficulties offering rationale for correct solutions	• Encourage students to describe the approach or conceptual underpinning even if they cannot mobilize an exacting procedure.
• Prefers performance based or portfolio type assessment to typical tests • Prefers comprehensive exams to quizzes and unit tests • More comfortable recognizing correct solutions than generating them	• May be overwhelmed when faced with multiple examples	• Consider a variety of assessment techniques • Allow oral presentations. • Do not always require exact solution but sometimes grade homework and tests only for correct approach. • Include some multiple choice items on tests.

Table 3

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Mathematical Learning Profiles and Differentiated Teaching Strategies

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Conclusion:

The *Child/World* System allows teachers to achieve an understanding of the dynamic interplay that affects a student's learning in mathematics. It leads to the delineation of specific Mathematical Learning Profiles. Extensive clinical investigations and classroom instruction, along with rigorous research efforts, have corroborated the presence of specific Mathematical Learning Profiles. Those learning profiles involve differences in development as well as preferences for models and preferences for approaches. Complicating the consideration of learning profiles in mathematics are more general memory and language issues that intrude on efforts in mathematical activities.

The understanding of Mathematical Learning Profiles helps teachers offer specific approaches and strategies that make use of students' areas of relative strengths, that minimize areas of vulnerability and that support areas of specific deficit, ensuring the comfortable participation and growth of all students in the mathematical arena. The importance to teachers of understanding Mathematical Learning Profiles is that they lead to the development of more effective learning strategies which, in turn, allow more students to experience success in the domain of mathematics.

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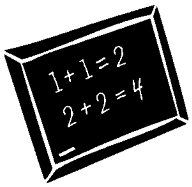
TRANSLATING LESSONS FROM RESEARCH INTO MATHEMATICS CLASSROOMS



Mathematics and Special Needs Students

By Douglas H. Clements

Too often students with learning disabilities receive limited mathematics instruction. This is due in part to special education teachers feeling uncomfortable teaching mathematics. This leads to an overemphasis on training skills. There are three reasons for this focus on skills. First, there is a major misconception that skill learning is the bedrock of mathematics, upon which all further mathematics must be built. Second, skills are easier to measure and teach. Third, teachers often believe that students' perceived memory deficits imply the need for constant repetition and drill.

Lessons from Research

Decades of research indicate that students can and should solve problems before they have mastered procedures or algorithms traditionally used to solve these problems (National Council of Teachers of Mathematics, 2000). If they are given opportunities to do so, their conceptual understanding and ability to transfer knowledge is increased (e.g., Carpenter, Franke, Jacobs, Fennema, & Empson, 1997).

Indeed, some of the most consistently successful of the reform curricula have been programs that

- build directly on students strategies;
- provide opportunities for both invention and practice;
- have children analyze multiple strategies;
- ask for explanations.

Research evaluations of these programs show that these curricula facilitate conceptual growth without sacrificing skills and also help students learn concepts (ideas) and skills while problem solving (Hiebert, 1999).

What is remarkable is that similar principles apply to students with learning disabilities. Many children classified as learning disabled can learn effectively with quality conceptually-oriented instruction (Parmar & Cawley, 1997). As the *Principles and Standards for School Mathematics* illustrates (National Council of Teachers of Mathematics, 2000), a balanced and comprehensive instruction, using the child's abilities to shore up weaknesses, provides better long-term results. For example, students may benefit less from intensive drill and practice and more from help searching for, finding, and using patterns in learning the basic number combinations and arithmetic strategies (Baroody, 1996).

Many of the lessons we have learned from research for general education students apply, with modifications of course, to students with special needs as well. A particularly important one is "less is more." That is, in mathematics and science, we have found that sustained time on fewer *key concepts* leads to greater overall student achievement in the long run. Compared to other countries that significantly outperform us on tests, U.S. curricula do not challenge students to learn important topics in depth (National Center for Education Statistics, 1996). We state many more ideas in an average lesson, but develop fewer of them, compared to other countries (Stigler & Hiebert, 1999). Thus, U.S. students would be better off focusing on in-depth study on fewer important concepts. Such an approach is critical with students with learning disabilities. They need to concentrate on mastering the key ideas, and these ideas are not arithmetic algorithms. Even proficient adults use relationships and strategies to produce basic facts. They tend not to use traditional paper-and-pencil algorithms when computing.

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> *Decades of research indicate that students can and should solve problems before they have mastered procedures or algorithms traditionally used to solve these problems.*

Another research lesson is that a variety of instructional materials is beneficial in meeting the needs of all students. Students who use manipulatives in their mathematics classes usually outperform those who do not (Driscoll, 1983; Greabell, 1978; Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1986). Manipulatives can be particularly helpful to students with learning disabilities.

Somewhat surprising, manipulatives do not necessarily have to be physical objects. Computer manipulatives can provide representations that are just as personally meaningful to students. Paradoxically, computer representations may even be more manageable, flexible, and extensible than their physical counterparts (Clements & McMillen, 1996). Students who use physical and software manipulatives demonstrate a greater mathematical sophistication than do control group students who use physical manipulatives alone (Olson, 1988). Good manipulatives are those that are meaningful to the learner, provide control and flexibility to the learner, have characteristics that mirror, or are consistent with cognitive and mathematical structures, and assist the learner in making connections between various pieces and types of knowledge. For example, computer software can dynamically connect pictured objects, such as base ten blocks, to symbolic representations. Computer manipulatives can play those roles. They help children generalize and abstract experiences with physical manipulatives.

Recommendations for Classroom Practice

Research provides several recommendations for meeting the needs of all students in mathematics education.

1. Keep expectations reasonable, but not low.

Low expectations are especially problematic because students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many non-white students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations. Expectations must be raised because "mathematics can and must be learned

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by all students" (NCTM, 2000). Raising standards includes increased emphasis on conducting experiments, authentic problem solving, and project-based learning (McLaughlin, Nolet, Rhim, & Henderson, 1999).

2. Patiently help students develop conceptual understanding and skills.

Students who have difficulty in mathematics may need additional resources to support and consolidate the underlying concepts and skills being learned. They benefit from multiple experiences with models and reiteration of the linkage of models with abstract, numerical manipulations.

Expand time for mathematics. In general, the traditional curriculum does not allow adequate time for the many instructional and learning strategies necessary for the mathematical success of learning disabled students (Lerner, 1997).

Students with disabilities may also need increased time to complete assignments. Finally, they may also benefit from more time or fewer examples on tests or from the use of oral rather than written assessments.

3. Build on children's strengths.

This statement often is little more than a trite pronouncement. But teachers can reinvigorate it when they make a conscientious effort to build on what children know how to do, relying on children's own strengths to address their deficits.

4. Build on children's informal strategies.

Even severely learning disabled children can invent quite sophisticated counting strategies (Baroody, 1996). Informal strategies provide a starting place for developing both concepts and procedures.

5. Develop skills in a meaningful and purposeful fashion.

Practice is important, but practice at the problem solving level is preferred whenever possible. Meaningful, purposeful practice gives us two for the price of one. Meaningless drill may actually be harmful to these children (Baroody, 1999; Swanson & Hoskyn, 1998).

6. Use manipulatives wisely.

Manipulatives can help learning disabled students learn both concepts and skills (Mastropieri, Scruggs, & Shiah, 1991). However, students should not learn to use manipulatives in a rote manner (Clements & McMillen, 1996). Make sure students explain what they are doing and link their work with manipulatives to underlying concepts and formal skills.

7. Use technology wisely.

It is important that all students have opportunities to use technology in appropriate ways so that they have access to interesting and important mathematical ideas. Access to technology must not become yet another dimension of educational inequity (NCTM, 2000). Computers can serve many purposes (Clements & Nastasi, 1992; Mastropieri et al., 1991; Pagliaro, 1998; Shaw, Durden, & Baker, 1998). Computers with voice-recognition or voice-creation software can offer teachers and peers access to the mathematical ideas and arguments developed by students with disabilities who would otherwise be unable to share their thinking. Computers can also serve as a valuable extension to traditional manipulatives that might be particularly helpful to special needs students (c.f. Weir, 1987).

Students should learn counting and arithmetical strategies but should also learn to use calculators for some purposes (Lerner, 1997). For students who can demonstrate a clear understanding of an operation, the calculator might be the primary means of computation (Parmar & Cawley, 1997).

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> *In general, the traditional curriculum does not allow adequate time for the many instructional and learning strategies necessary for the mathematical success of learning disabled students.*

8. Make connections.

Integrate concepts and skills. Help children link symbols, verbal descriptions, and work with manipulatives. Use every possible social situation to provide meaningful situations for mathematical problem solving opportunities. (Baroody, 1999; Parmar & Cawley, 1997).

9. Adjust instructional formats to individual learning styles or specific learning needs.

Formats might include modeling, demonstration, and feedback; guiding and teaching strategies; mnemonic strategies for learning number combinations; and peer mediation (Gersten, 1985; Lerner, 1997; Mastropieri et al., 1991). Use projects and games to help the teacher guide learning, rather than relying solely on "telling" (Baroody, 1999). The traditional sequence of direct teacher explanations, strategy instruction, relevant practice, and feedback and reinforcement is often effective, but the potential of students to learn through problem solving should not be ignored. Too often, direct instruction approaches squeeze out other possibilities. Use direct instruction only when students are unable to invent their own strategies. In all cases, help them make strategies explicit (Kame'enui & Carnine, 1998).

10. Emphasize statistics, geometry, and measurement as well as arithmetic topics (Parmar & Cawley, 1997).

All students need access to varied topics in mathematics. Topics beyond arithmetic are increasingly important in our day-to-day lives.

Overall, solve problems, encourage reasoning, and use modeling. With patience and support, these processes are also in the reach of most children.

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